

Personnel selection using type-2 fuzzy ahp method

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Abstract

Classical Analytic Hierarchy Process (AHP) developed by Saaty (1980) uses a crisp scale. Humans are more successful in comparing criteria or alternatives using the expressions like "between 3 and 4" or "about 3". In the literature, many type-1 fuzzy AHP methods have been developed. The authors of this paper have first time developed the type-2 fuzzy AHP method to handle the uncertainty in defining membership functions. Personnel selection is a multi criteria problem with many conflicting main and sub criteria. We select the best candidate among three candidates who apply for a position in a manufacturing firm by using type-2 fuzzy AHP method.

Introduction

Personnel selection is the process used to hire individuals. Although the term can apply to all aspects of the process, the most common meaning focuses on the selection of workers. In this respect, selected prospects are separated from rejected applicants with the intention of choosing the person who will be the most successful and make the most valuable contributions to the organization. Personnel selection is a multi criteria problem with many conflicting main and sub criteria. Classical Analytic Hierarchy Process (AHP) developed by Saaty (1980) uses a crisp scale. Humans are more successful in comparing criteria or alternatives using the expressions like "between 3 and 5" or "about 3". For this aim, fuzzy AHP was developed by some researchers in the past (Buckley, 1985; Chang, 1996; Laarhoven and Pedrycz, 1983). These fuzzy AHP methods are based on type-1 fuzzy sets.

Type-2 fuzzy sets generalize type-1 fuzzy sets and systems so that more uncertainty can be handled. While excessive arithmetic operations are needed with type-2 fuzzy sets with respect to type-1's, type-2 fuzzy sets can handle the uncertainty in defining membership functions. In type-1 fuzzy sets, each element has degree of membership which is described with a membership function valued in the interval $[0, 1]$ (Zadeh, 1965). The concept of a type-2 fuzzy set was introduced by Zadeh(1975) as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set; hence, they are useful for incorporating linguistic uncertainties, e.g., the words that are used in linguistic knowledge can mean different things to different people (Karnik and Mendel, 2001).

In this paper, a multicriteria personnel selection will be made based on type-2 fuzzy AHP method. The considered criteria are experience, job knowledge, health, adaptability, potential, and age. We compare three candidates who apply for a position in a business firm.

Type-1 Fuzzy AHP Methods

In Table 1, we present the existing type-1 fuzzy AHP methods in the literature.

Table 1. The comparison of different fuzzy AHP methods (Büyüközkan et al., 2004)

Sources	The main characteristics of the method	Advantages (A) and disadvantages (D)
Van Laarhoven and Pedrycz (1983)	Direct extension of Saaty's AHP method with triangular fuzzy numbers Lootsma's logarithmic least square method is used to derive fuzzy weight and fuzzy performance scores.	(A) The options of multiple experts can be modeled in the reciprocal matrix (D) There is not always a solution to linear equations (D) The computational requirement is tremendous, even for a small problem (D) It allows only triangular fuzzy numbers to be used
Buckley (1985)	Extension of Saaty's AHP method with trapezoidal fuzzy numbers Uses the geometric mean method to derive fuzzy weights and performance scores	(A) It is easy to extend to the fuzzy case (A) It guarantees a unique solution to the reciprocal comparison matrix (D) The computational requirement is tremendous
Boender et al. (1989)	Modifies van Laarhoven and Pedrycz's method Present a more robust approach to the normalization of the local priorities	(A) The options of multiple experts can be modeled (D) The computational requirement is tremendous
Chang (1996)	Synthetical degree values Layer simple sequencing Composite total sequencing	(A) The computational requirement is relatively low (A) It follows the steps of crisp AHP. It doesn't involve additional operations (D) It allows only triangular fuzzy numbers to be used
Cheng (1996)	Builds fuzzy standards Represent performance scores by membership functions	(A) The computational requirement is tremendous (D) Entropy is used when probability distribution is known (D) The method is based on both probability and possibility measures
Zeng et al. (2007)	Uses arithmetic averaging method to get performance scores. Extension of Saaty's AHP method with different scales contains triangular, trapezoidal, and crisp numbers.	(A) It follows the steps of crisp AHP (A) The options of multiple experts can be modeled (A) There is a flexibility of using different scales (D) The computational requirement is tremendous when there are much expert's judgements.

Operations with Type-2 Fuzzy Sets

A trapezoidal interval type-2 fuzzy set is illustrated as

$$\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = \left((\tilde{a}_{11}^U, \tilde{a}_{12}^U, \tilde{a}_{13}^U, \tilde{a}_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (\tilde{a}_{11}^L, \tilde{a}_{12}^L, \tilde{a}_{13}^L, \tilde{a}_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \right)$$

where \tilde{A}_1^U and \tilde{A}_1^L are type-1 fuzzy sets, $\tilde{a}_{11}^U, \tilde{a}_{12}^U, \tilde{a}_{13}^U, \tilde{a}_{14}^U, \tilde{a}_{11}^L, \tilde{a}_{12}^L, \tilde{a}_{13}^L, \tilde{a}_{14}^L$ are the reference points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_1$, $H_j(\tilde{A}_1^U)$ denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function \tilde{A}_1^U , $1 \leq j \leq 2$, $H_j(\tilde{A}_1^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function \tilde{A}_1^L , $1 \leq j \leq 2$, $H_1(\tilde{A}_1^U) \in [0,1]$, $H_2(\tilde{A}_1^U) \in [0,1]$, $H_1(\tilde{A}_1^L) \in [0,1]$, $H_2(\tilde{A}_1^L) \in [0,1]$, and $1 \leq i \leq n$. Two trapezoidal interval type-2 fuzzy sets are given by $\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = (\tilde{a}_{11}^U, \tilde{a}_{12}^U, \tilde{a}_{13}^U, \tilde{a}_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (\tilde{a}_{12}^L, \tilde{a}_{13}^L, \tilde{a}_{14}^L, \tilde{a}_{12}^L;$

$$H_1(\tilde{A}_1^L), H_2(\tilde{A}_2^L)) \text{ and } \tilde{\tilde{A}}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = (\tilde{a}_{21}^U, \tilde{a}_{22}^U, \tilde{a}_{23}^U, \tilde{a}_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), (\tilde{a}_{21}^L,$$

$$\tilde{a}_{22}^L, \tilde{a}_{23}^L, \tilde{a}_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L))$$

The addition operation between the trapezoidal interval type-2 fuzzy sets is defined as follows:

$$\begin{aligned} \tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) = \\ & \left((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U, \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L + \right. \\ & \left. a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L, \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))) \right) \end{aligned} \tag{1}$$

The subtraction operation between the trapezoidal interval type-2 fuzzy sets is defined as follows:

$$\begin{aligned} \tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \ominus (\tilde{A}_2^U, \tilde{A}_2^L) = \\ & \left((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U, \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L - \right. \\ & \left. a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L, \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))) \right) \end{aligned} \tag{2}$$

The multiplication operation between the trapezoidal interval type-2 fuzzy sets is defined as follows:

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) = \\ & \left((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U, \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L \times \right. \\ & \left. a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L, \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))) \right) \end{aligned} \tag{3}$$

A constant is multiplied by a type-2 fuzzy set as

follows: $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = (\tilde{a}_{11}^U, \tilde{a}_{12}^U, \tilde{a}_{13}^U, \tilde{a}_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (\tilde{a}_{11}^L, \tilde{a}_{12}^L, \tilde{a}_{13}^L, \tilde{a}_{14}^L;$

$H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))$) and the crisp value k is defined as follows:

$$k\tilde{A}_1 = \left((k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times \right. \\ \left. a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \right), \tag{4}$$

$$\begin{aligned} \frac{\tilde{A}_1}{k} &= \\ & \left(\left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \right), \left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times \right. \right. \\ & \left. \left. a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L) \right) \right), \end{aligned} \tag{5}$$

Where $k > 0$.

Steps of Type-2 Fuzzy AHP

In this section, Buckley's type-1 fuzzy AHP method will be modified by using interval type-2 fuzzy sets. The procedure of this fuzzy AHP method is explained as follows:

Step 1: Fuzzy pair wise comparison matrices among all the criteria in the dimensions of the hierarchy system are constructed. The result of the comparisons is constructed as fuzz ypair wise comparison matrices (\tilde{A}) as following;

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \cdots & 1 \end{bmatrix} \tag{6}$$

where

$$1/\tilde{a} = \left(\left(\frac{1}{a_{14}^U}, \frac{1}{a_{13}^U}, \frac{1}{a_{12}^U}, \frac{1}{a_{11}^U}; H_1(a_{12}^U), H_2(a_{13}^U) \right), \left(\frac{1}{a_{24}^L}, \frac{1}{a_{23}^L}, \frac{1}{a_{22}^L}, \frac{1}{a_{21}^L}; H_1(a_{22}^L), H_2(a_{23}^L) \right) \right)$$

The linguistic variables and their triangular and trapezoidal interval type-2 fuzzy scales which can be used in interval type-2 fuzzy AHP are given in Table 2.

Table 2. Definition and interval type 2 fuzzy scales of the linguistic variables

Linguistic variables	Triangular Interval Type-2 fuzzy scales	Trapezoidal Interval Type-2 fuzzy scales
Absolutely Strong (AS)	(7.5,9,10.5;1) (8.5,9,9.5;0.9)	(7,8,9,9;1,1) (7.2,8.2,8.8,9;0.8,0.8)
Very Strong (VS)	(5.5,7,8.5;1)(6.5,7,7.5;0.9)	(5,6,8,9;1,1) (5.2,6.2,7.8,8.8;0.8,0.8)
Fairly Strong (FS)	(3.5,5,6.5;1)(4.5,5,5.5;0.9)	(3,4,6,7;1,1) (3.2,4.2,5.8,6.8;0.8,0.8)
Slightly Strong (SS)	(1.5,3,4.5;1)(2.5,3,3.5;0.9)	(1,2,4,5;1,1) (1.2,2.2,3.8,4.8;0.8,0.8)
Exactly Equal (E)	(1,1,1;1)(1,1,1;1)	(1,1,1,1;1,1) (1,1,1,1;1,1)
If factor <i>i</i> has one of the above linguistic variables assigned to it when compared with factor <i>j</i> , then <i>j</i> has the reciprocal value when compared with <i>i</i> .	Reciprocals of above	Reciprocals of above

Step 2: The consistency of each fuzzy pair wise comparison matrix is examined. Assume $A = [a_{ij}]$ is a positive reciprocal matrix and $\tilde{A} = [\tilde{a}_{ij}]$ is a fuzzy positive reciprocal matrix. If the result of the comparisons of $A = [a_{ij}]$ is consistent, then it can imply that the result of the comparisons of $\tilde{A} = [\tilde{a}_{ij}]$ is also consistent. In order to check the consistency of the fuzzy pairwise comparison matrices, the proposed DTriT or DTraT approach is used.

$$DTriT = \frac{\frac{(u_U - l_U) + (m_U - l_U)}{3} + l_U + \alpha \left[\frac{(u_L - l_L) + (m_L - l_L)}{3} + l_L \right]}{2} \tag{7}$$

$$DTraT = \frac{\frac{(u_U - l_U) + (\beta_U \cdot m_U - l_U) + (\alpha_U \cdot m_U - l_U)}{4} + l_U + \left[\frac{(u_L - l_L) + (\beta_L \cdot m_L - l_L) + (\alpha_L \cdot m_L - l_L)}{4} + l_L \right]}{2} \tag{8}$$

Step 3: The geometric mean of each row is calculated and then the fuzzy weights are computed by normalization.

The geometric mean of each row \tilde{r}_i is calculated as follows;

$$\tilde{r}_i = [\tilde{a}_{i1} \otimes \dots \otimes \tilde{a}_{in}]^{1/n}$$

Where

$$\begin{aligned}
 \sqrt[n]{\tilde{a}_{ij}} = & \left(\left(\sqrt[n]{a_{ij1}^U}, \sqrt[n]{a_{ij2}^U}, \sqrt[n]{a_{ij3}^U}, \sqrt[n]{a_{ij4}^U}; H_1^u(a_{ij}), H_2^u(a_{ij}) \right), \right. \\
 & \left. \left(\sqrt[n]{a_{ij1}^L}, \sqrt[n]{a_{ij2}^L}, \sqrt[n]{a_{ij3}^L}, \sqrt[n]{a_{ij4}^L}; H_1^l(a_{ij}), H_2^l(a_{ij}) \right) \right) \\
 & (9)
 \end{aligned}$$

The fuzzy weight \tilde{w}_i of the i^{th} criterion is calculated as follows;

$$\tilde{w}_i = \tilde{r}_i \otimes [\tilde{r}_1 \oplus \dots \oplus \tilde{r}_i \oplus \dots \oplus \tilde{r}_n]^{-1} \tag{10}$$

where

$$\begin{aligned}
 \frac{\tilde{a}_{ij}}{\tilde{b}_{ij}} = & \left(\frac{a_1^u}{b_4^u}, \frac{a_2^u}{b_3^u}, \frac{a_3^u}{b_2^u}, \frac{a_4^u}{b_1^u}, \min(H_1^u(a), H_1^u(b)), \min(H_2^u(a), H_2^u(b)) \right) \\
 & \left(\frac{a_1^l}{b_4^l}, \frac{a_2^l}{b_3^l}, \frac{a_3^l}{b_2^l}, \frac{a_4^l}{b_1^l}, \min(H_1^l(a), H_1^l(b)), \min(H_2^l(a), H_2^l(b)) \right)
 \end{aligned}$$

Step 4: The fuzzy weights and fuzzy performance scores are aggregated as follows:

$$\tilde{U}_i = \sum_{j=1}^n \tilde{w}_j \tilde{r}_{ij} \tag{11}$$

where \tilde{r}_{ij} is the fuzzy utility of alternative i ; \tilde{w}_j is the weight of the criterion j , and \tilde{r}_{ij} is the performance score of alternative i with respect to criterion j .

Step 5: The classical AHP method’s procedure is applied to determine the best alternative.

Personnel Selection Using Type-2 Fuzzy AHP

The hierarchy of the considered personnel selection problem is given in Figure 1.

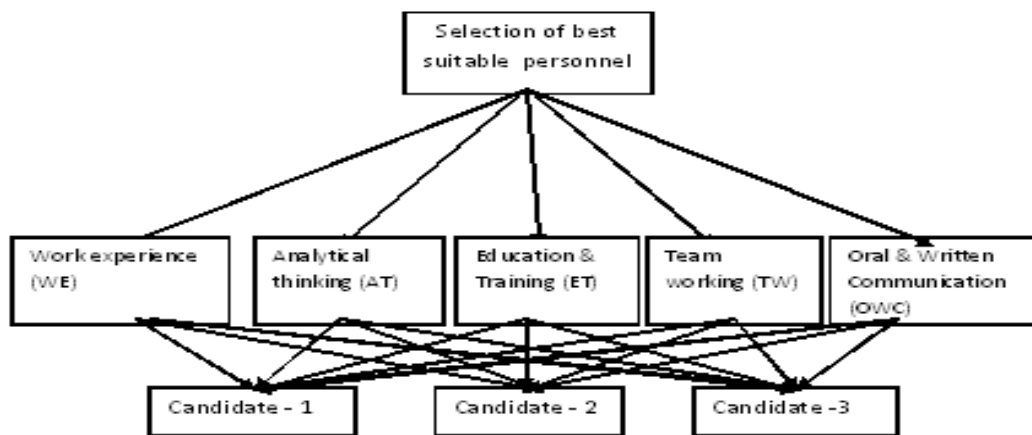


Figure 1. The hierarchy of the personnel selection problem

Tables 3-8 show the epairwise comparison matrices using linguistic evaluations.

Table 3. Pairwise comparison matrix for criteria

With respect to the goal	WE	AT	ET	TW	OWC
WE	E	FS	1/SS	VS	VS
AT		E	1/FS	VS	FS
ET			E	VS	FS
TW				E	1/VS
OWC					E

Table 4. Pairwise comparison of candidates with respect to WE

With respect to WE	C-1	C-2	C-3
C-1	E	VS	FS
C-2		E	1/SS
C-3			E

Table 5. Pairwise comparison of candidates with respect to AT

With respect to AT	C-1	C-2	C-3
C-1	E	FS	1/VS
C-2		E	1/VS
C-3			E

Table 6. Pairwise comparison of candidates with respect to ET

With respect to ET	C-1	C-2	C-3
C-1	E	1/FS	SS
C-2		E	VS
C-3			E

Table 7. Pairwise comparison of candidates with respect to TW

With respect to TW	C-1	C-2	C-3
C-1	E	SS	E
C-2		E	1/FS
C-3			E

Table 8. Pairwise comparison of candidates with respect to OWC

With respect to OWC	C-1	C-2	C-3
C-1	E	E	1/SS
C-2		E	1/SS
C-3			E

Using the scale given in Table 2 and applying the steps of type-2 fuzzy AHP method, the following results have been obtained:

The weights obtained from Table 3 are given in Table 9.

Table 9. Weights of the Criteria

Criteria	Fuzzy weights	Defuzzified weights
WE	(0.168,0.236,0.461,0.709;1,1)(0.180,0.252,0.428,0.642;0.8,0.8)	0.338007
AT	(0.036,0.049,0.092,0.137;1,1)(0.038,0.052,0.086,0.126;0.8,0.8)	0.067823
ET	(0.209,0.330,0.659,0.931;1,1)(0.232,0.356,0.617,0.866;0.8,0.8)	0.460437
TW	(0.036,0.047,0.083,0.118;1,1)(0.038,0.050,0.078,0.109;0.8,0.8)	0.061457
OWC	(0.039,0.053,0.098,0.144;1,1)(0.042,0.056,0.091,0.132;0.8,0.8)	0.072277

The weights obtained from Table 4 are as follows:

Table 10. Weights of Alternatives with respect to WE

Alternatives	Fuzzy weights	Defuzzified weights
C-1	(0.428,0.568,0.933,1.216;1,1)(0.455,0.598,0.888,1.150;0.8,0.8)	0.717308
C-2	(0.048,0.062,0.112,0.178;1,1)(0.051,0.065,0.104,0.159;0.8,0.8)	0.090358
C-3	(0.090,0.136,0.256,0.362;1,1)(0.100,0.146,0.241,0.336;0.8,0.8)	0.192334

The other sets of weights are obtained in the same way. They are given in Table 11:

Table 11. Defuzzified Weights of Alternatives with respect to AT, ET, TW, and OWC

Alternatives	With respect to AT	With respect to ET	With respect to TW	With respect to OWC
C-1	0.186064	0.204994	0.395120	0.207799
C-2	0.065824	0.684514	0.131817	0.207799
C-3	0.748112	0.110492	0.473063	0.584402

The final weights of the alternatives after the combination of the obtained weights above are given in Table 12.

Table 12. Final Weights of Alternatives

Alternatives	Fuzzy weights	Crisp Weights
C-1	(1.117,1.844,4.380,7.126;1,1)(1.247,2.018,4.012,6.400;0.8,0.8)	0.384242
C-2	(0.978,1.758,4.415,7.057;1,1)(1.116,1.945,4.042,6.381;0.8,0.8)	0.377955
C-3	(0.664,1.094,2.668,4.559;1,1)(0.742,1.197,2.429,4.041;0.8,0.8)	0.237803

Candidate 1 should be selected.

Conclusion

Personnel selection is a multi criteria selection problem with many conflicting criteria under fuzziness. There are numerous criteria considered in the personnel selection publications. Among these, the most appropriate criteria for the firm should be handled. Humans are more successful in assigning linguistic values rather than numerical ones. The fuzzy set theory provides useful tools to incorporate these linguistic evaluations into a multi criteria methodology. We used the type-2 fuzzy AHP method developed by us to evaluate alternative

candidates who apply for a job. For further research, our proposed method, which is based on Buckley's type-1 fuzzy AHP, may be compared with other type-2 fuzzy AHP methods, which may be based on Laarhoven and Pedrycz's (1983) or Chang's (1996) methods

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